

Cyclic Prefix in OFDM

Frequency Selective Channel

$$y(n) = h(0)x(n) + h(1)x(n-1) \\ + h(2)x(n-2) + \dots + h(L-1)x(n-L+1).$$

↓
Model for Freq. Selective
channel.

2 Consecutive OFDM Symbols.

$$\tilde{X}_0, \tilde{X}_1, \dots, \tilde{X}_{N-1}$$

↓ IFFT

$$\tilde{x}(0) \tilde{x}(1) \dots \tilde{x}(N-1)$$

Transmit samples
of OFDM Sym 1

$$X_0, X_1, \dots, X_{N-1}$$

↓ IFFT

$$x(0) x(1) \dots x(N-1)$$

Transmit samples
of OFDM Sym 2

output for $x(0)$

$$y(0) = h(0)x(0) + h(1)\tilde{x}(N-1) + h(2)\tilde{x}(N-2) \\ + \dots + h(L-1)\tilde{x}(N-L+1)$$

$$y(0) = h(0)x(0) + \underline{h(1)\tilde{x}(N-1)} \\ + \underline{h(2)\tilde{x}(N-2) + \dots + h(L-1)\tilde{x}(N-L+1)}$$

Inter Block Interference

(IBI) } Avoid
IBI

Cyclic Prefix:

$$X_0 \quad X_1 \quad X_2 \quad \dots \quad X_{N-1}$$

↓ IFFT

$$\underbrace{x(N-L) \dots x(N-2) x(N-1)}_{\text{Prefix of } L \text{ Samples from Tail}} \quad x(0) x(1) x(2) \dots x(N-1) \quad \underbrace{\hspace{1cm}}_{L \text{ samples.}}$$

Prefix of L
Samples from Tail

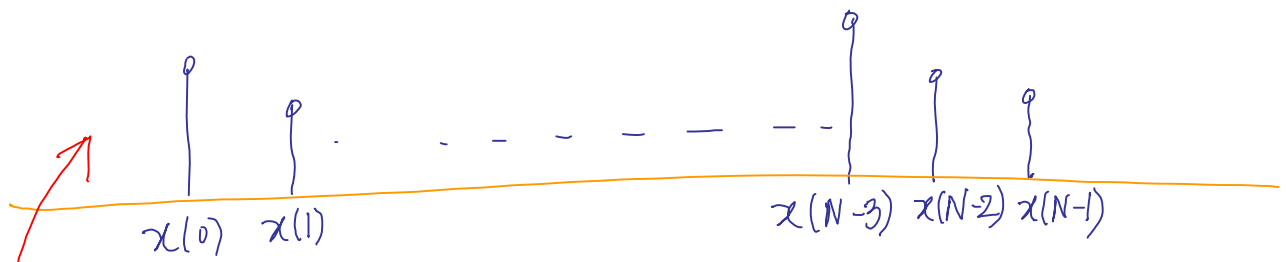
$$\underline{x(N-L) \dots x(N-1) x(0) x(1) \dots x(N-1)}$$

Prefix

$$y(0) = h(0)x(0) + h(1)x(N-1) + h(2)x(N-2) \\ + \dots + h(L-1)x(N-L+1)$$

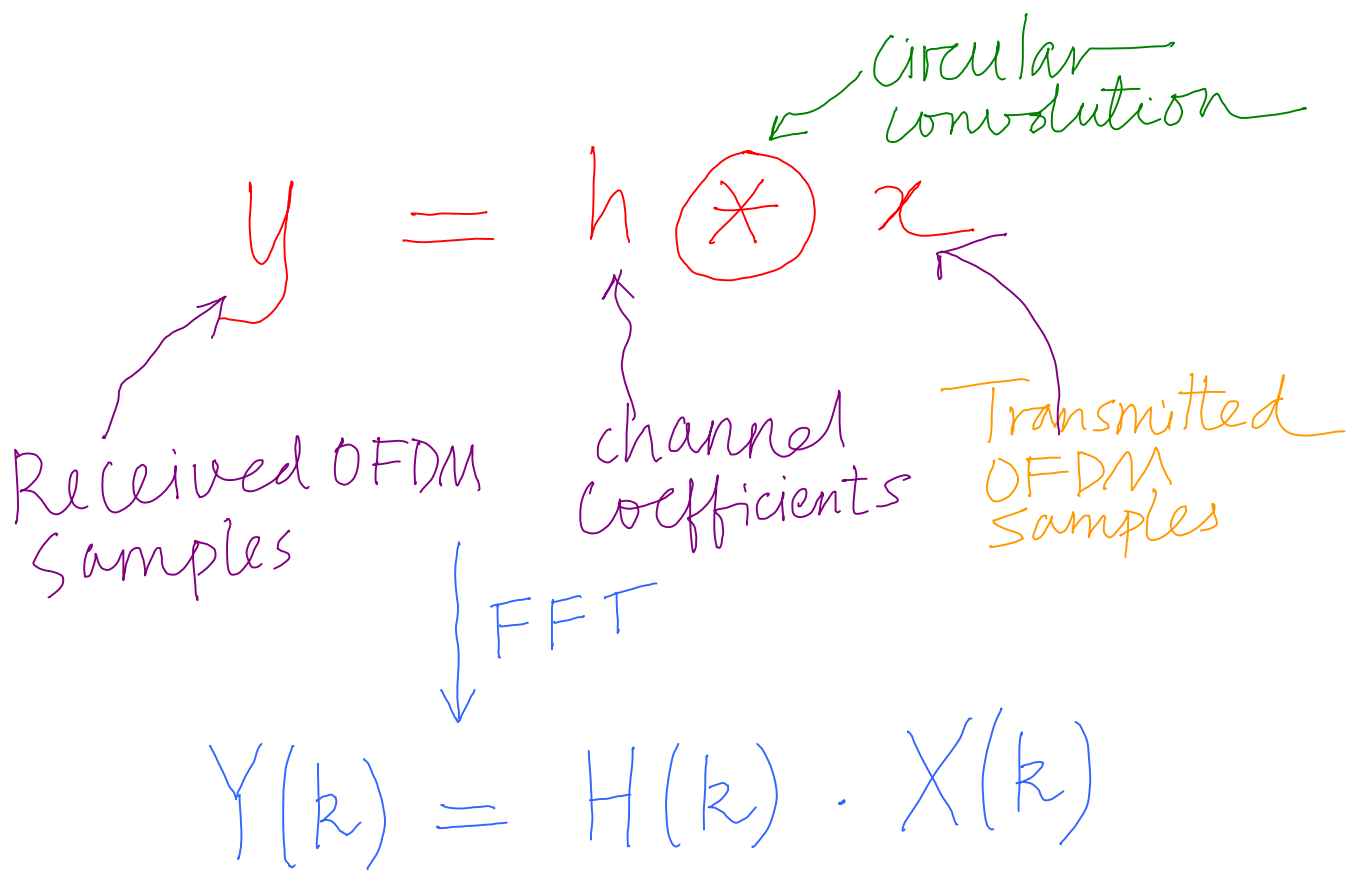
$$y(1) = h(0)x(1) + h(1)x(0) + h(2)x(N-1) \\ + \dots + h(L-1)x(N-L+1)$$

$$\begin{array}{l}
 n=1 \quad h(1) \quad h(0) \quad \quad \quad h(4) \quad h(3) \quad h(2) \\
 \hline
 n=0 \quad h(0) \quad \quad \quad h(3) \quad h(2) \quad h(1) \\
 \hline
 \end{array}$$



Output OFDM signal is a circular convolution between $h \circledast x$

Addition of cyclic Prefix (CP)
has resulted in a circular
convolution at the output
of the OFDM system.



$$Y(k) = H(k) \cdot X(k)$$

↑

k^{th} DFT coefficient
of OFDM Received
signal $[y(0) y(1) \dots y(N-1)]$

↘ k^{th} DFT coeff
 $[x(0) x(1) \dots x(N-1)]$

k^{th} DFT/FFT
Coefficient of N pt FFT of
 $[h(0) h(1) \dots h(L-1) \underbrace{0 \ 0 \ \dots \ 0}_{N-L \text{ zeros}}]$

$$X_0 \ X_1 \ X_2 \ \dots \ X_{N-1}$$

↓ IFFT

$$x(0) \ x(1) \ \dots \ x(N-1)$$

↓ FFT

$$X(0) \ X(1) \ \dots \ X(N-1)$$

$$X(0) = X_0 \quad X(1) = X_1 \quad \dots \quad X(N-1) = X_{N-1}$$

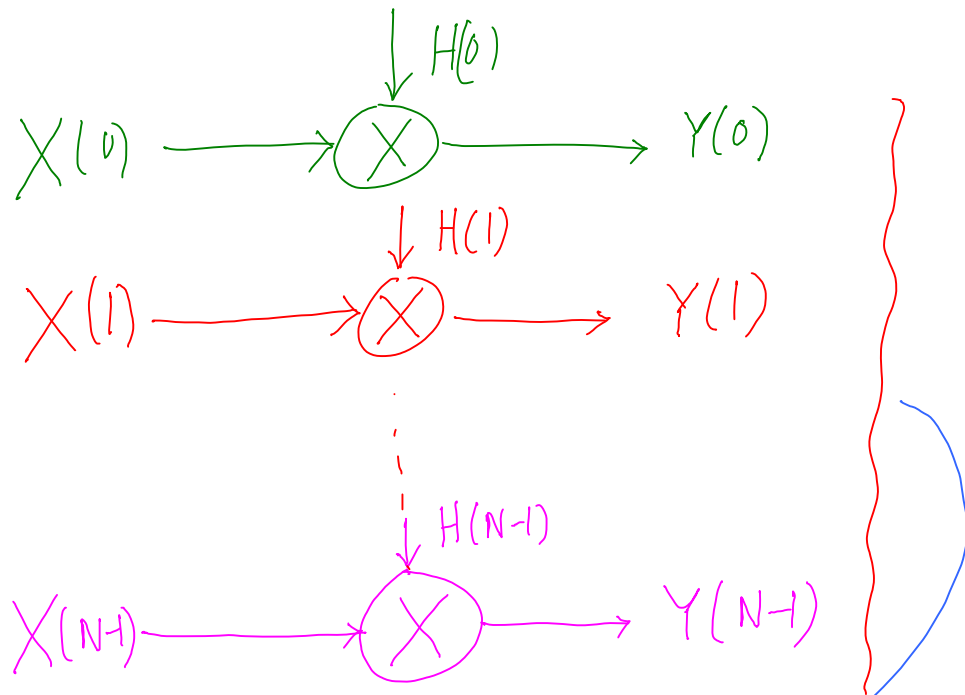
$$Y(k) = H(k) \cdot X(k)$$

Symbol on k^{th}
subcarrier

channel
coefficient
of k^{th} subcarrier

Symbol on k^{th}
subcarrier

Each subcarrier is
experiencing Flat Fading.



OFDM has converted
FS channel into N Flat
Fading Channels.